MANDURAH CATHOLIC COLLEGE

Matrices
Test 5 2017
Section 1 Calculator-Free

MATHEMATICS SPECIALIST
Year 11

NAME: Solutions

TEACHER: ______________________

RESULT CF: _______/16 RESULT CA: _______/27

TOTAL: _______/43 PERCENTAGE: _______%

TIME ALLOWED FOR THIS PAPER

Working time for paper:
Section 1 = 15 minutes
Section 2 = 30 minutes
Total Time = 45 minutes

MATERIALS REQUIRED/RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE TEACHER

This Question/Answer Booklet, SCSA Formula Sheet

TO BE PROVIDED BY THE STUDENT

Standard Items: Pens, pencils, eraser or correction tape, ruler, protractor.

IMPORTANT NOTE TO STUDENTS

No other items may be taken into the classroom. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the classroom. If you have any unauthorised material with you, hand it to the teacher BEFORE reading any further.

Instructions to Students

1. **ALL** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet.

3. **SHOW ALL YOUR WORKING CLEARLY**. Your working should be sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks.

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5. It is recommended that you **do not use pencil**, except in diagrams.

Question 1 **[2, 2, 2, 3 = 9 marks]**

Consider the matrices $A = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & -5 \end{bmatrix}$.

(a) It is possible to form the product of all four matrices. State the dimensions of the resulting product.

$ABDC = 2 \times 3$

✓correct product

✓dimensions

(b) Determine the matrix $\frac{1}{2}DC$.

$\frac{1}{2}DC = \frac{1}{2}\begin{bmatrix} 4 & -5 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 4 & -10 & 6 \end{bmatrix}$ ✓multiplies matrices correctly

$= \begin{bmatrix} 2 & -5 & 3 \end{bmatrix}$ ✓works it through

(c) Determine the inverse of matrix A.

$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$

✓Det A

✓inverse matrix

(d) Clearly show use of matrix algebra to solve the system of equations $2x - 3y + 3 = 0$ and $4y = 2x + 2$.

$\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ ✓creates equ.

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ ✓inverses

$= \frac{1}{2}\begin{bmatrix} -6 \\ -2 \end{bmatrix}$

$\therefore x = -3, y = -1$ ✓solves.

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Question 2 **[2, 2, 3 = 7 marks]**

(a) Determine the value(s) of a for which the matrix $\begin{bmatrix} a & a \\ 3 & 2a \end{bmatrix}$ is singular.

$\det = 2a^2 - 3a$ ✓

$0 = 2a^2 - 3a$

$0 = a(2a - 3)$

$a = 0$ or $a = \frac{3}{2}$ ✓

(b) The non-singular matrix B is such that $[-3 \quad 2]\times B = [8 \quad 3]$ and $[2 \quad 6]\times B = [10 \quad 4]$.

(i) Use these results to show that $[-1 \quad 8]\times B = [18 \quad 7]$.

$[-3 \quad 2] \times B + [2 \quad 6] \times B = [8 \quad 3] + [10 \quad 4]$ ✓ Adds

$([-3 \quad 2] + [2 \quad 6]) \times B = [18 \quad 7]$

$[-1 \quad 8] \times B = [18 \quad 7]$ ✓ Factorises and result

(ii) Determine $[2 \quad 1]\times B^{-1}$.

$[2 \quad 6] \times B - [-3 \quad 2] \times B = [10 \quad 4] - [8 \quad 3]$ ✓ subtracts

$([2 \quad 6] - [-3 \quad 2]) \times B = [2 \quad 1]$ ✓ Factorises

$[5 \quad 4] = [2 \quad 1] \times B^{-1}$ ✓ solves.

END OF SECTION ONE

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Additional working space

Question number:___________

MANDURAH CATHOLIC COLLEGE

Matrices
Test 5 2017
Section 2 Calculator-Assumed

MATHEMATICS SPECIALIST
Year 11

NAME: ______________________________

TEACHER: ______________________________

RESULT CA: _______/27

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TO BE PROVIDED BY THE STUDENT

Standard Items: Pens, pencils, eraser or correction tape, ruler, protractor, CAS/scientific calculator, one side of one A4 page notes.

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Question 3 **[2, 2, 3 = 7 marks]**

Transformation A is an anti-clockwise rotation about the origin of 90° and matrix $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(a) Describe the transformation represented by matrix B.

Enlargement by factor 2 parallel to x-axis. ✓

Enlargement by factor 3 parallel to y-axis. ✓

(b) Determine the coordinates of the point $P(-15, -11)$ following transformation A and then transformation B.

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -15 \\ -11 \end{bmatrix} = \begin{bmatrix} 22 \\ -45 \end{bmatrix}$$

✓ correct order in multiplication.

$P' = (22, -45)$ ✓

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(c) Following transformation B and then transformation A, point Q is transformed to point $Q'(12, 7)$.

Determine the single matrix that will transform Q' back to Q and hence determine the coordinates of point Q.

Let $Q = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$ ✓ Transformation matrix

$\therefore$ matrix that will transform Q' back to Q is

$$\begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}$$ ✓ Inverse of mapping back.

Hence, $Q = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} 3.5 \\ -4 \end{bmatrix}$

$Q(3.5, -4)$ ✓ Q

3

Question 4 **[3 marks]**

Given that the point $(-4, 6)$ is rotated clockwise around the origin to $(5.86, 4.21)$, find the angle at which it is rotated clockwise to the nearest degree.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 5.86 \\ 4.21 \end{bmatrix}$$ ✓ sets up eqn.

$\therefore -4\cos\theta - 6\sin\theta = 5.86$

Using CAS

$\theta = 271.96^\circ$ ✓ solves

Hence, clockwise rotation $= 88^\circ$ ✓ clockwise rotation.

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Question 5 **[1, 2, 2 = 5 marks]**

The four points $O(2,1), A(2,3), B(5,3)$ and $C(5,1)$ form a rectangle.

(a) Find the area of the rectangle $OABC$.

OABC Area = 6 units^2 ✓ must have units^2

(b) If O, A, B and C are transformed to the points O', A', B' and C' by the matrix $M = \begin{bmatrix} 3 & 4 \\ 10 & 5 \end{bmatrix}$, find the area of the quadrilateral $O'A'B'C'$.

Det $M = 15 - 40$
$= -25$ ✓

$\therefore$ Area $O'A'B'C' = 25 \times 6$
$= 150$ units^2 ✓

(c) Has the quadrilateral $OABC$ been reflected in its transformation? Explain.

Yes as the determinant is negative ✓ ✓

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Question 6 **[3, 1, 2 = 6 marks]**

Consider the curve with equation $y = f(x)$. The curve has a maximum point at A (-1, 3) and a minimum point at B (4, -7). The curve $y = f(x)$ is mapped onto the curve $y = g(x)$ by a transformation represented by the matrix $\boldsymbol{T} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(a) Find the coordinates of the images of the points A and B under **T**.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 3 & -7 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -3 & 7 \end{bmatrix}$$ ✓ Transform

Hence $A = (-1, 3)$, $B = (4, 7)$ ✓✓ A, B

(b) Describe the effect the transformation represented by T has on the graph of $y = f(x)$.

Reflection about the x-axis. ✓

(c) Find the coordinates of the maximum and minimum points on the curve $y = g(x)$.

Max $= B'(4, 7)$ ✓

Min $= A(-1, 3)$ ✓

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Question 7 **[3 marks]**

If $\boldsymbol{A}^2 - 3\boldsymbol{A} + \boldsymbol{I} = 0$, show that $\boldsymbol{A}^{-1} = 3\boldsymbol{I} - \boldsymbol{A}$

$A^2 - 3A + I = 0$

$A^2 - 3A = -I$ ✓ I to RHS

$A(A - 3I) = -I$ ✓ Factorises

$A - 3I = -A^{-1}I$

$3I - A = A^{-1}$ ✓

 3

Question 8 **[2, 1 = 3 marks]**

A carpenter runs a business making three different models of cubby house for children. Each cubby house is made using four different sizes of treated pine timber. The number of metres of each size of timer required for each cubby house is shown below:

$$\begin{array}{r} \\ \text{Cubby A} \\ \text{Cubby B} \\ \text{Cubby C} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Poles} & \text{Decking} & \text{Framing} & \text{Sheeting} \end{array} \\ \begin{bmatrix} 3 & 30 & 20 & 40 \\ 4 & 35 & 25 & 60 \\ 6 & 40 & 30 & 70 \end{bmatrix} \end{array}$$

(a) The carpenter receives an order for 3 Cubby As, 1 Cubby B and 2 Cubby Cs. By using a matrix method, determine the total number of poles, decking, framing and sheeting the carpenter needs for the order.

$$\begin{bmatrix} 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 30 & 20 & 40 \\ 4 & 35 & 25 & 60 \\ 6 & 40 & 30 & 70 \end{bmatrix} = \begin{bmatrix} 25 & 205 & 145 & 320 \end{bmatrix}$$

They will need 25 poles, 205 decking pieces, 145 framing pieces. and 320 sheeting pieces.

(b) If the poles cost \$4 per metre, the decking \$2 per metre, the framing \$3 per metre and the sheeting \$1.50 per metre, use a matrix method to determine how much would it cost the carpenter to make the order from part (a)?

$$\begin{bmatrix} 25 & 205 & 145 & 320 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 3 \\ 1.5 \end{bmatrix} = \$1425$$

END OF ASSESSMENT

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Additional working space

Question number:___________